Coherent sources and robustness

Sverre Holm

UNIVERSITY OF OSLO

Covariance matrix

- Single signal: $\mathbf{y} = A \cdot \mathbf{s} + \mathbf{n}$
  - A – signal amplitude
  - **s** – signal vector
  - **n** – noise
- Covariance, 1 signal :
  - $\mathbf{R} = \sigma_n^2\, \mathbf{I} + A^2\, \mathbf{e} \cdot \mathbf{e}'$
  - Eigenvalues: σ^2_n (x M-1), $\sigma^2_n + MA^2$
  - Largest eigenvector $\mathbf{v} = M^{0.5}\, \mathbf{s}$ (signal vector)
  - Known array geometry $\Rightarrow$ find direction and amplitude

2010.05.06 2

UNIVERSITY OF OSLO

2 incoherent signals

- Two incoherent signals:
  - $\mathbf{R} = \sigma_n^2 \mathbf{I} + A_1^2 e_1 \cdot e_1' + A_2^2 e_2 \cdot e_2'$
  - The signal vectors, $\mathbf{S}=[e_1\ e_2]$ are linearly indep. for different directions (and properly sampled array)
  - Noise subspace: M-2 noise eigen-values/vectors
  - Signal+noise subspace: 2 orthogonal eigenvectors $\mathbf{V}_{s+n} = [v_1, v_2]$ (Hermitian matrix)
  - Span a subspace that contains s_1 and s_2: $\mathbf{V}_{s+n}\ \mathbf{T} = \mathbf{S}$,
  - $\mathbf{T}$ = transf. matrix (unique, may be hard to find)

Multiple signals

- Multiple signals:
  - $\mathbf{R} = \mathbf{K_n} + \mathbf{SCS}'$
  - $\mathbf{C}$ is intersignal coherence
- Examples:
  - Spatially white noise: $\mathbf{K_n} = \sigma_n^2\ \mathbf{I}$
  - Two incoherent signals: $\mathbf{C} = \text{diag}(A_1^2, A_2^2)$ – diagonal matrix

Coherent signals

- Two perfectly coherent signals:

$$C = \begin{bmatrix} A^2 & -A^2 \\ -A^2 & A^2 \end{bmatrix}$$

Resulting covariance matrix:

- **R** = **K**$_n$ + **SCS'** $= \sigma_n^2$**I** + A^2 $[e_1 - e_2] \cdot [e_1 - e_2]$
  - M-1 eigenvalues of size σ^2_n, one of size $\sigma^2_n + MA^2$
  - New eigenvector, but as signal vectors **s** are linearly independent, a linear combination of them is not a new signal
  - The new eigenvector is related to the signal vectors, but does not correspond to a physical direction, θ
  - Cannot find T from **V**$_{s+n}$ **T** = **S**

Coherent signals and spatial smoothing

- Spatial smoothing
- Compromise between
  - … smoothing to avoid the effect of coherent signals
  - … and loss of resolution due to subaperture smaller than physical aperture

Robustness

- The more ”tuned” an algorithm is, the more sensitive it is to deviations from assumptions
- Assumed form of the signal vector implies perfect knowledge of:
  - Sensor positions
  - Sensor gains
  - Sensor phase
    - changes if speed of propagation in medium is incorrect

Robust Constrained Optimization

- Minimum variance beamforming:
  1. Minimize **w’Rw** with respect to **w**
  2. Subject to **e’w** = 1 – unity gain, desired direction
- Robustness criterion 1:
  2. Subject to **(e+**δ**)’w** = 1 and $|\delta|^2 \le \varepsilon^2$
    - δ represents errors in signal vector
- Robustness criterion 2:
  2. Subject to **e’w** = 1 and $|w|^2 \le \beta^2$
    - β represents a limit on the weight vector’s norm
    - Not directly related to robustness, but …

Robust Constrained Optimization

- Both cases ⇒ add a scaled identity matrix to covariance estimate: **R** → **R** + ε **I**
  - ⇔ Regularization in linear algebra
  - ⇔ Diagonal loading in array processing
- Value of ε depends on criterion and is signal dependent
  - Du, Yardibi, Li, Stoica, Review of user parameter-free robust adaptive beamforming algorithms, Digital Signal Processing, 2009
- Simple solution used by us: ε = δ·tr{R}/L, L= sub. ap